# Chapter 1: Introduction

*Nothing can exceed the wondrous beauty of Zion . . .*
*In the nobility and beauty of the sculptures, there is no comparison . . .*
*There is an eloquence to their forms*
*which stirs the imagination with a singular power,*
*and kindles in the mind a glowing response.*
(Clarence Dutton, writing of Zion National Park)

### *1.1 The Thesis*

Natural forms, their manifestation, their changing with time, their immense variety and pervasiveness, constitute and motivate much of conscious experience. In this dissertation we develop methods that employ inner structure in the design and representation of natural forms. These methods are presented for use in computer graphics.

Although 'form,' like the word 'shape,' refers to outward appearance, it also suggests inner structure [Mish 1984]. We interpret inner structure as the medial component or components of a form, whether or not they physically exist. For example, the medial component of an ellipsoid is a line segment. We define the 'skeleton' of an object as the collection of its medial components.

We propose the following thesis: the design of many natural forms can be effected with the skeleton. That is, the process of shape design involves the discernment of inner structure within a real or imagined object, and the use of inner structure promotes an effective and interactive design environment.

In this dissertation, the skeleton is the principal design unit. We contend it is a powerful and intuitive means to abstract existing forms and to define new ones. It is a simple construct, easily understood and manipulated, yet able to generate geometrically complex surfaces. Accordingly, the ability to discern or imagine the skeleton within a natural shape is fundamentally important to the designer.

Many natural forms result from a particular organization of inner structure. Vertebrate animals depend on bones for mechanical support and on muscles for movement. Thus, guides for artists typically illustrate bones, musculature, and tissue [Bridgman 1960].[1*] Similarly, inner structure is valuable to an artist's understanding of plants and trees [Cole 1951]. Indeed, the inner structure of a plant is so apparent that often there is little embellishment between its skeleton and its visualization [Aono and Kunii 1984], [Smith 1984]. Nonetheless, the relationship of inner structure to a plant's surface may be complex, as we demonstrate in a later chapter examining the botanical leaf.

A skeleton is not always the most warranted representation for a shape, particularly a synthetic one. For example, a brick is more easily represented by width, height, and depth than by a skeleton. Or, folds in fabric are better defined by tensions and pressures transmitted along the surface than as a covering of some inner structure. Nonetheless, the skeleton appears to be a suitable abstraction for many natural forms, including inorganic ones. For example, the undulations in a sand dune can be abstracted as a smooth cover over an undulating skeleton. Thus, we contend that a computer mediated design system predicated on the skeleton, able to reflect the aesthetic concerns of a designer and able to represent a diversity of structures, can provide a powerful means for geometric modeling, artistic expression, and appreciation of Nature.

In Nature we may find virtually every arrangement and combination of elementary forms. The diversity is so great that in this dissertation we cannot scrutinize all natural forms. For reasons given in the next chapter, we restrict our consideration to those natural forms whose shape is smooth.

* Notes appear at the end of each chapter.

*1.2 Literature*

Natural form is not studied as a separate scientific discipline; rather, relevant literature is found within several disciplines, including anatomy, archaeology, biomorphology, botany, paleontology, and zoology. In biology, the literature of morphology discusses biological shape, usually in its relation to biological function. In the literature of mathematics, geometric modeling, and engineering, there is considerable analysis of the geometry of form, including curves, surfaces, and solids. Few studies concern natural shapes with the exception of fractals [Mandelbrot 1983]. In other fields, such as medicine and architecture, there is little concern for the compact representations that are useful in design.

The best introduction to the design of natural form is found in the natural sciences, particularly the works of [Hertel 1966], [Stevens 1974], [Thompson 1961], [Wainwright 1988], and [Whyte 1968]. Other sources may deserve mention, but nowhere is found a more erudite examination of natural form and its mathematical representation than in [Thompson 1961]. To date, this seminal work is the most complete survey of natural form and contains an abundance of original observations, derivations, and mathematical representations.

None of these sources, however, develops its material to include the realistic synthesis or animation of a shape. For these operations we turn primarily to the literature of computer graphics, which has established a rich history of representation, visualization, and animation of selected natural forms. To a lesser extent, we find similar reports in the literature of biomathematics. Although a survey of these works is beyond the scope of this dissertation, we note relevant publications when developing specific techniques.

Skeletons have previously been used to define shape [Burtnyk 1976]. In computer graphics the most commonly discussed aspect of the skeleton is its articulation; the covering of the skeleton is usually relegated to a straightforward process that often yields insufficiently realistic surfaces. Complex covering mechanisms are discussed in [Forsey 1991] and [Bloomenthal and Wyvill 1990]; in this dissertation we attempt to develop these mechanisms more fully. There is a substantial literature concerning the visual discernment of shape, which relates to skeletal definition. Relevant texts are [Duda and Hart 1973] and [Uttal 1988].

## *1.3 Organization*

This dissertation is organized into eight chapters. The first two are introductory, presenting arguments that shapes are best designed from the 'inside out.' In later chapters we examine technical problems associated with this approach, at times comparing them to alternate, non-skeletal methods. The computer graphicist interested in technical issues only may wish to move directly to these chapters.

### *1.3.1 The Design of Shape*

In chapter 2 we discuss the context within which form has significance. We examine the extent to which skeletons may be employed to realistically represent shape and we argue that implicit surfaces provide a useful covering of the skeleton. As observed in [Thompson 1961], the need for science to understand causation should not impede observations of Nature and correlative inferences. In this dissertation, our aim is not to determine why a shape is as it is, but to offer insights as to how it may be mimicked. This objective is well suited for computer graphics, which devotes substantial effort to the representation and visualization of imagined shapes.

An analysis of shape design might include functional views, system views, and user views of the design environment. We do not, unfortunately, offer a user view because we have not implemented a real-time, interactive interface for the design methods discussed in this dissertation. We do, however, offer extensive functional views throughout several chapters and a system view in the appendix.

### *1.3.2 Implicit Surfaces and Non-Manifold Surfaces*

Surface representation may be separated into three principal classes: explicit, implicit, and parametric [Faux and Pratt 1979]. A two-dimensional curve, for example, can be represented explicitly as $y = f(x)$, parametrically as $(x, y) = (X(t), Y(t))$, and implicitly as $f(x, y) = 0$. A prominent example of a parametric surface is the bicubic patch [Catmull 1974]; a prominent example of an implicit surface is the quadric surface [Foley *et al*. 1990]. Implicit surfaces tend to describe volumes, which we have found useful in the modeling of natural forms. Their application within a design environment will be discussed in the next chapter.

Because our emphasis is design and not visualization, we desire a concrete representation of shape, and there is no more general, concrete representation than that of polygons. A polygonal representation, which we regard as equivalent to the *boundary* or *evaluated* form [Mortensen 1985], is a well-established data structure, a *lingua franca* of computer graphics and geometric modeling. Thus, we devote substantial portions of chapter 3 to implicit surface polygonization.

In chapter 4 we discuss the combination of implicit and parametric surfaces as a design methodology. Parametric surfaces are useful for explicit tangent or positional control and for representing sheets rather than volumes. We develop a new methodology that represents a parametric surface except where it is enclosed within an implicit volume. This method can produce non-manifold implicit surfaces. In Chapter 4 we extend polygonization to include this form of geometric model, and we provide several examples. Not until chapter 7, however, do we apply non-manifold techniques to the modeling of a natural form.

### *1.3.3 Blends*

In chapter 5 we review the representation of smooth surfaces, settling on an implicit method that utilizes convolution. We examine particular blends, bulge prevention, and *n*-way branching, and we consider artistic, computational, and geometric aspects of the relation between skeleton and surface. In chapter 5 we lay the mathematical foundations necessary for the examples given in chapters 6 and 7. We also consider the addition of detail through the process of texture mapping. Only in chapter 7, however, do we discuss a specific method to generate surface detail.

### *1.3.4 Examples*

For those objects for which skeletal representation is practical, we must consider the relationship of skeleton to form. This includes the girth of component volumes surrounding skeletal elements, the method of blending component volumes, and the extent to which these methods realistically model particular forms. These questions are considered in general throughout the dissertation and particular answers are offered for two natural objects: the human hand and the maple leaf. These two examples not only demonstrate new techniques, but prompted their development. We do not yet know how easily other natural forms may be modeled with the design techniques presented in this dissertation.

In chapter 6 the one-dimensional skeletal elements employed in chapter 5 are extended to two dimensions to produce a model of the human hand. A new method to avoid bulges is presented. Issues of blend avoidance and surface detail are also discussed. In chapter 7 we apply the techniques of non-manifold implicit surfaces to the design of a botanical leaf that combines implicitly defined veins and a parametrically defined leaf blade.

## *1.4 Contributions*

In this dissertation we develop new techniques for the application of three-dimensional convolution to curves and surfaces, with an emphasis on bulge-free and crease-free blends. A new presentation and analysis of convolution surfaces is given, along with a new analysis of filter kernels, separability, and the bulge problem. A fundamental result of this dissertation is the establishment of constraints on the skeleton such that seamless, bulge-free surfaces are produced.

Other new techniques include the interpolation of union and convolution surfaces, implicitly defined cross-sections, surface-volume blends, leaf venation, and polygon triangulation.

This dissertation offers new presentations of the problems of shape design, metamorphosis, implicit surface polygonization, manifold and non-manifold surfaces, reference frames, generalized cylinders, distance surfaces, and implicit surface texture. These presentations include new analyses of reference frames, implicit surface ramification, and surface detail.

### *1.5 Notes*

1. Indeed, Leonardo was an early proponent of dissection studies.

*Biological diversity makes a world of difference.*

(The United States National Park Service)